Covert Path Distortion Correction for E.O. Sensor Systems.

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Abstract

A method has been derived for extending signal processing algorithms, which have previously been used to correct RF communications channels for the effects of multi-path, into a form that may be suitable for correcting distorted images from EO sensors subjected to atmospheric refractive path perturbations.

Keywords: *image distortion, EO sensor, image deblurring, eigenvector, EVA, EVI*

Introduction

High-resolution observation of distant objects/targets using high magnification EO sensors is fundamentally constrained by, amongst other factors, the refractive distortion of the signal propagation path. Conceptually this problem can be compared to the RF communications domain where transmitted signals can suffer multipath interference by reflections from buildings or land masses, or refraction in the upper atmosphere.

The major difference between the two situations is that for a communications channel the received signal is a one dimensional value as a function of time, whereas an EO image is a two dimensional function of space. Figures 1 and 2 illustrate the problems in the two domains.

In this precursor study one particular electronic signal processing technique, developed to solve the RF multi-path problem, has been extended to improve the performance of EO imager systems. If successful this technique will provide a solution to compensate for image distortion caused by refractive index perturbations which is covert, simple and low cost. The technique can be used for off-line image enhancement, or else it can potentially be embedded in a real-time processing device. Since it does not require additional optical devices it can be retro-fitted to existing equipment.

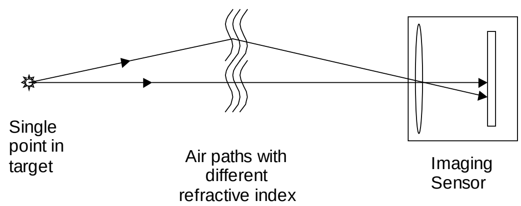

Figure 1 Simplified image path distortion

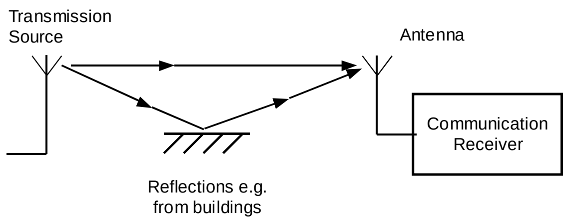

Figure 2 Communications channel multi-path distortion

This technique aims to enhance sensor performance through:

- Extending sensor range performance without requiring active illumination (long range & covert).
- Detecting lower signature features within the image.

- Implementation using digital signal processing technology (affordable and robust).
- Simplified, lower mass/volume solution extending applications to e.g. small UAVs, man-portable systems etc.

Existing One Dimensional Algorithms

The original one dimensional algorithms which have been used successfully for communications systems are known as "EVA" (EigenVector Algorithm for Blind Equalisation) and the EVI algorithm (EigenVector approach to blind Identification). They were developed by researchers at the University of Bremen [1]. The EVA algorithm is used to determine the coefficients of an equaliser function which approximates the inverse of the path distortion function. The EVI algorithm estimates the coefficients of the path distortion function itself.

Figure 3 below represents the combination of the multi-path signals via a set of mixing coefficients *h(k)* assuming a sampled data stream *d(k)*. (A real-world signal is also likely to suffer from added noise, but for clarity this is not shown). The simplest process to recover approximately the transmitted data is to use a similar mixing process on the received data samples, using a linear equaliser with coefficients *e(k)* which achieves an inverse of the path mixing process (see Figure 4).

The path can be considered to be equivalent to a finite impulse response (FIR) filter. An important point to note is that even for a finite length path *h(k)* the ideal *e(k)* has infinite length. In practice a finite length *e(k)* is assumed, however in difficult cases it may need to be many times the length of *h(k)* to achieve reasonable performance. Because of these problems, it may be preferable to estimate the path coefficients *h(k)* directly. An equaliser is still required, but may not be necessarily a linear one.

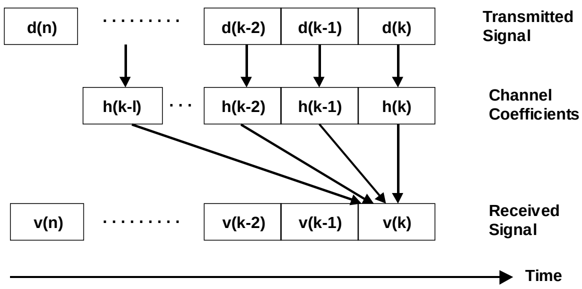

Figure 3 Signal mixing due to channel

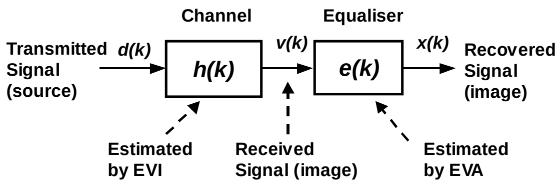

Figure 4 Recovery of undistorted signal using equaliser

Mathematical Outline of Algorithms

Algorithms for blind signal equalisation and channel estimation rely on statistical properties of the received signal *v(k)*. There is a basic separation, discussed in [2], between methods which use second order statistics and those using higher, usually fourth, order statistics such as EVA and EVI. which do not simply calculate statistics based on comparing the signal (or image), denoted *v(k)*, with itself. The EVA and EVI work by iterating the equaliser *e(k)* and evaluating fourth-order cross-cumulant statistics of *v(k)* combined with an estimate of the recovered signal, denoted *y(k)*, over the set of data samples *k* (time or pixel co-ordinate). In particular a matrix $\mathbf{C}_4^{yv}$ is formed having elements calculated as:

$$
\begin{aligned}
c_4^{yv}(i_1,i_2) = \\
&\mathbf{E}\{y^*(k)\, v^*(k+i_1)\, y(k)\, v(k+i_2)\} \\
-&\mathbf{E}\{y^*(k)\, y(k)\}\,\mathbf{E}\{v^*(k+i_1)\, v(k+i_2)\} \\
-&\mathbf{E}\{y^*(k)\, v^*(k+i_1)\}\,\mathbf{E}\{y(k)\, v(k+i_2)\} \\
-&\mathbf{E}\{y^*(k)\, v(k+i_2)\}\,\mathbf{E}\{v^*(k+i_1)\, y(k)\}
\end{aligned}
$$

where **E{}** denotes expectation and * the complex conjugate. The expectations can be estimated using a straightforward averaging process.

EVA Algorithm Details

For the EVA algorithm the indexes $\boldsymbol{i}_1$ and $\boldsymbol{i}_2$ span the range of the elements of the equaliser filter e(k). In order to form the recovered signal *y(k)* an initial estimate of the filter is required. In the absence of any other information this can be set to a transparent filter, i.e. all elements zero except for the central element equal to 1.

A covariance matrix $\mathbf{R}_{\mathbf{vv}}$ is also defined indexed the same way, with each element r_{ij} given by:

$$r_{ij} = \mathbf{E}\{v^*(k-i)v(k-j)\}$$

The EVA algorithm then solves the following standard eigenvector equation:

$$\mathbf{R}_{vv}^{-1}\mathbf{C}_4^{yv}\mathbf{e} = \lambda\,\mathbf{e}$$

The coefficients for an "improved" equaliser e(k) is given by the principal eigenvector i.e. that which has the largest absolute valued eigenvalue. Note this has an ambiguity in phase, which must be resolved by other means.

EVI Algorithm Details

The EVI algorithm follows on from the EVA making use of e(k) the approximate equaliser and making an assumption of the size of h(k) the path distortion function. An additional vector order *n* is used in the algorithm where *n* is a superset of the order of the path distortion function *h*. A value of $n=4*h$ is believed to be most appropriate.

A modified cumulant matrix $\tilde{\mathbf{C}}_4^{yv}$ is formed in a similar way to $\mathbf{C}_4^{yv}$ except it is only the centrally positioned [(2*q*+1) by (*n*+1)] sub-matrix of the [(*n*+1) by (*n*+1)] matrix $\mathbf{C}_4^{yv}$.

The covariance matrix $\mathbf{R}_{\mathbf{vv}}$ is also defined as for the EVA but now with dimension [(*n*+1) by (*n*+1)].

$\tilde{\mathbf{R}}_{inv}$ is then defined to be a banded matrix of dimension [(*n*+1) by (2*q*+1)] with *n*+1 symmetrically placed non-zero bands whose values are given by the elements of the vector **r** of dimension *n*+1 which is the solution of the equation

$$\mathbf{R}_{vv}\mathbf{r} = \mathbf{i}$$

where **i** also has dimension *n*+1 and has a single central element 1, all other elements being 0.

Finally the estimated channel vector **h** is derived from $\hat{\mathbf{h}}$ the principal eigenvector solution to:

$$\tilde{\mathbf{C}}_4^{yv}\tilde{\mathbf{R}}_{inv}\hat{\mathbf{h}} = \lambda\hat{\mathbf{h}}$$

Extension to Two or More Dimensions

During this study a plausible and internally consistent two dimensional generalisation of the EVA and EVI algorithms has been devised which may be applied to at least some variants of the problem of interest. If simplified to the one dimensional case it reproduces the standard one dimensional algorithms.

The process consists of defining a number of two (or more) dimensional sets of offsets from a central co-ordinate, which are indexed in a strict order thus defining a mapping from the index to the offset. The offsets are those of the various filter coefficients as follows:

Q: the (unknown) path distortion point-spread function.

QL: the approximate equaliser function.

QN: an extended version of the estimated path distortion function (EVI only).

In the one dimensional case, the size of the equaliser *QL* and the EVI map *QN* need to be integer multiples of the *Q* map. This is now reinterpreted as that a multiple of a set is the set of all sums of elements of the summed sets, which in this case are the same set *Q*. Using the nomenclature *Q2* to represent the set equal to *Q+Q*, *Q4* for *Q2+Q2* and so on, it will probably be most convenient to use *Q2*, *Q4* or *Q6* for *QL*, and *Q4* or *Q6* for *QN*.

Some example two dimensional maps are shown in Figure 5 and Figure 6 below. The values shown in each box represent the relative offsets (Y,X) from the particular central pixel.

	-1,0	
0,-1	0,0	0,1
	1,0	

		-2,0		
	-1,-1	-1,0	-1,1	
0,-2	0,-1	0,0	0,1	0,2
	1,-1	1,0	1,1	
		2,0		

Figure 5 Simple examples of a set *Q* (left) and corresponding set *Q2* (right).

	-2,-1	-2,0	-2,1	
-1,-2	-1,-1	-1,0	-1,1	-1,2
0,-2	0,-1	0,0	0,1	0,2
1,-2	1,-1	1,0	1,1	1,2
	2,-1	2,0	2,1	

		-4,-2	-4,-1	-4,0	-4,1	-4,2		
	-3,-3	-3,-2	-3,-1	-3,0	-3,1	-3,2	-3,3	
-2,-4	-2,-3	-2,-2	-2,-1	-2,0	-2,1	-2,2	-2,3	-2,4
-1,-4	-1,-3	-1,-2	-1,-1	-1,0	-1,1	-1,2	-1,3	-1,4
0,-4	0,-3	0,-2	0,-1	0,0	0,1	0,2	0,3	0,4
1,-4	1,-3	1,-2	1,-1	1,0	1,1	1,2	1,3	1,4
2,-4	2,-3	2,-2	2,-1	2,0	2,1	2,2	2,3	2,4
	3,-3	3,-2	3,-1	3,0	3,1	3,2	3,3	
		4,-2	4,-1	4,0	4,1	4,2		

Figure 6 Larger examples of *Q* and *Q2*

It is important to note that the number of elements in *Q2* is *not* twice the number of elements in *Q* and so on. For example in the second example *Q* has 21 elements while *Q2* has 69 and *Q4* will have 249.

Two Dimensional Algorithm Outline

In order to meet one of the fundamental requirements of the algorithm, the image data must firstly be biased such that the total mean value of all pixels is zero. The sets *Q*, *QL* and *QN* must be defined and the initial equaliser coefficients stored in a vector of length equal to the number of elements in *QL*.

The EVA algorithm then proceeds by forming the cumulants as described in the preceding sections, at each valid pixel position in the image (index *k*). The cumulants at this position in the image are determined using indexes ***i*** or ***j*** which are obtained from the set *QL*. The contributions to the cumulants are evaluated and then the whole set is moved on to the next pixel by updating *k* and the next contributions are evaluated and so on until all the valid contributions have been made. The pixels being referenced by a set (mapping function) is illustrated in Figure 7.

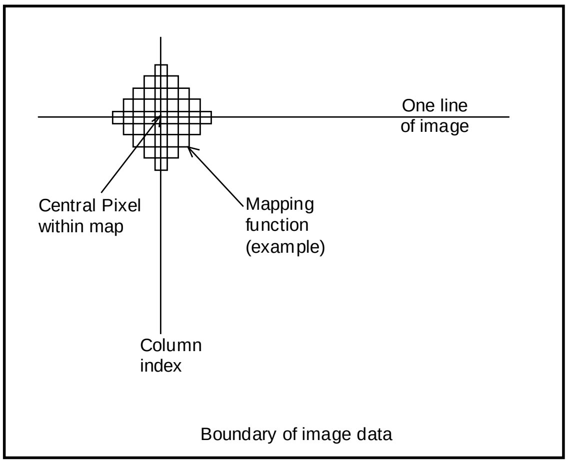

Figure 7 selection of pixels referenced by a set mapping function

The EVI algorithm uses a similar extension, using the additional set *QN* to map the estimated extended path coefficients.

Apart from the symmetric non-causal nature of this revised presentation of the EVA and EVI algorithms, the key property of the invertable mappings defined by the indexing of the sets *Q*, *QL* and *QN* established in this study is that for any other such invertable mapping (in effect a permutation of the indexing of the offsets in the sets *Q*, *QL* and *QN*) the relationship to the one dimensional case is consistent, i.e. the rows and columns of all matrices and vectors permute consistently.

Results from Study

To confirm the basic operation of the EVA algorithm a Matlab script has been produced. This was firstly applied to a test image consisting of a set of uniformly distributed random valued pixels, corrupted by a relatively simple path distortion (point spread function) of modest scale (Figure 8 to Figure 10) and to a photographic image similarly distorted (Figure 9 to Figure 13). These results show that the EVA algorithm produces a reasonably effective equaliser function.

Recommendations for future work

The next phase should be to investigate the algorithm with various test cases considering the following points:

Effect of scene characteristics

The EVA/EVI algorithms make certain assumptions about the statistical nature of the image data viz. the samples are statistically independent, zero mean and non-Gaussian with non-zero kurtosis. This means the image must not be simply white noise but it must have a reasonable amount of detail. The efficacy of the algorithm needs to be checked using several different types of test image.

Variation of distortion over whole image

The algorithms assume that the distortion is constant over the whole data set. This may be a constraint with real images where the image distortion could vary significantly from one region of the total image to another.

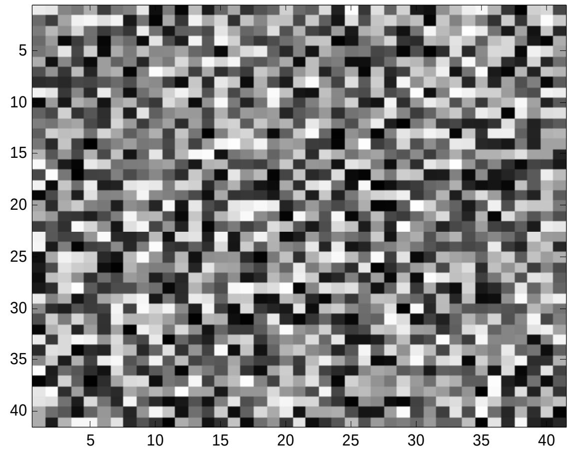

Figure 8 Section of undistorted random source image data

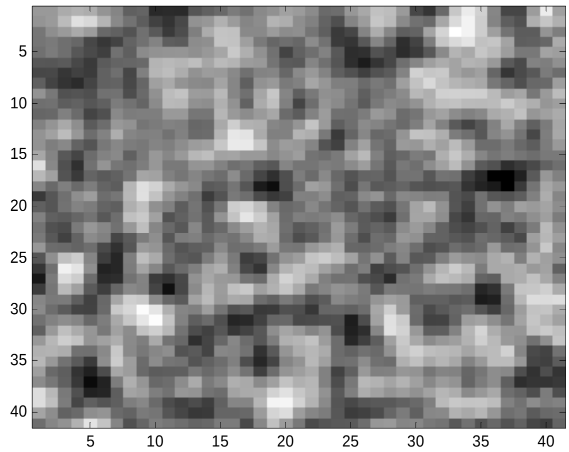

Figure 9 Image subjected to 13-element point spread function

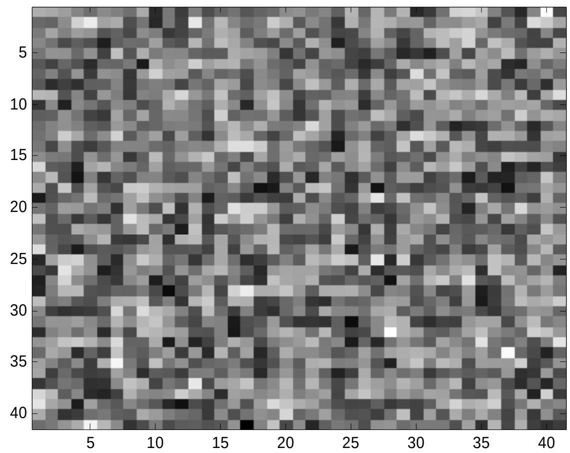

Figure 10 Recovered random image using EVA algorithm

Figure 11 Section of undistorted real image

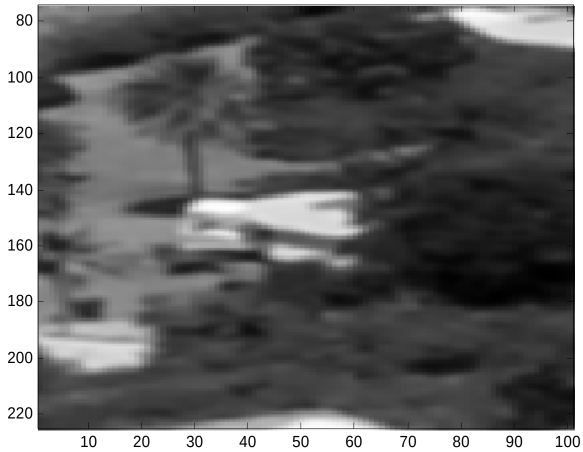

Figure 12 Image subjected to 13-element point spread function

Figure 13 Recovered image using EVA algorithm

Use of the time dimension

So far the algorithm has only been considered in application to single frame images. It would be possible to include the data from multiple frames directly in the two dimensional algorithm leading to a simple averaging effect. The trade-off between improving the robustness and tracking the change of distortion from frame to frame needs to be assessed.

Conclusions

At this stage a Matlab script has been created to execute the two dimensional EVA algorithm and has given confidence of the approach being successful. The performance of the algorithm should now be evaluated to determine the relationships between image segment size, filter length, noise resistance and efficacy for a variety of image cases.

References

[1] University of Bremen, Department of Communications Engineering, Research and Publications, http://www.ant.uni-bremen.de/research

[2] B. Jelonnek et. al., “Generalised Eigenvector Algorithm for Blind Equalization”, Elsevier Signal Processing, Vol. 61, No. 3, pp. 237-264, September 1997.

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